## Exercise 68

(a) Use a graph to estimate the absolute maximum and minimum values of the function to two decimal places.

(b) Use calculus to find the exact maximum and minimum values.

$$f(x) = x - 2\cos x, \quad -2 \le x \le 0$$

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**Solution**

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x - 2\cos x) \\ &= \frac{d}{dx}(x) - 2\frac{d}{dx}(\cos x) \\ &= (1) - 2(-\sin x) \\ &= 1 + 2\sin x \end{aligned}$$

Set $f'(x) = 0$ and solve for $x$.

$$1 + 2\sin x = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = -\frac{\pi}{6} + 2\pi n \quad \text{or} \quad x = -\frac{5\pi}{6} + 2\pi n, \quad n = 0, \pm 1, \pm 2, \ldots$$

Only $x = -\pi/6$ is within the interval $-2 \le x \le 0$. Evaluate the function here.

$$f\left(-\frac{\pi}{6}\right) = \left(-\frac{\pi}{6}\right) - 2\cos\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6} - \sqrt{3} \approx -2.25565 \qquad \text{(absolute minimum)}$$

Evaluate the function at the endpoints.

$$f(-2) = (-2) - 2\cos(-2) = -2(1 + \cos 2) \approx -1.16771 \qquad \text{(absolute maximum)}$$

$$f(0) = (0) - 2\cos(0) = -2$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $-2 \le x \le 0$.

The graph below illustrates these results.